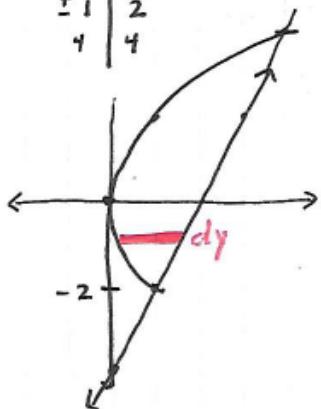


AP Calculus AB

Average Value of a Function

1) $y^2 = 4x$ $y = 2x - 4$

| x | y |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 4 | 4 |



$\frac{1}{4}y^2 = x$

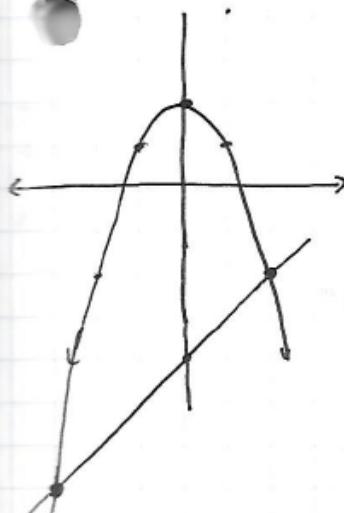
$\frac{1}{2}y + 2 = x$

Area = $\int_{-2}^4 \left[\frac{1}{2}y + 2 - \frac{1}{4}y^2 \right] dy$

= $\left[\frac{1}{4}y^2 + 2y - \frac{1}{12}y^3 \right] \Big|_{-2}^4$

= $\left[4 + 8 - \frac{64}{12} \right] - \left[1 - 4 + \frac{8}{12} \right]$

2) $y = 2 - x^2$ $y = x - 4$



$2 - x^2 = x - 4$

$-x^2 - x + 6 = 0$

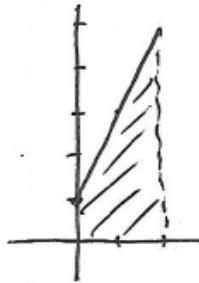
$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, x = 2$

$$\begin{aligned} \text{Area} &= \int_{-3}^2 [2 - x^2 - (x - 4)] dx \\ &= \int_{-3}^2 [-x^2 - x + 6] dx \\ &= \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x \right] \Big|_{-3}^2 \\ &= \left[-\frac{8}{3} - 2 + 12 \right] - \left[9 - \frac{9}{2} - 18 \right] \end{aligned}$$

3) $y = 2x + 1$ on $[0, 2]$



Area = $\int_0^2 [2x + 1] dx$

= $\left[x^2 + x + C \right] \Big|_0^2$

= $[4 + 2]$

= 6

$$4) \frac{1}{3} \int_{-1}^2 [5x^4 + 3x^2] dx$$

$$\frac{1}{3} \left[x^5 + x^3 + C \right] \Big|_{-1}^2$$

$$\frac{1}{3} [(32+8) - (-1-1)]$$

$$\frac{1}{3}(42) = \boxed{14}$$

$$5) \frac{1}{\pi} \int_0^\pi \sin x dx$$

$$\frac{1}{\pi} \left[-\cos x + C \right] \Big|_0^\pi$$

$$\frac{1}{\pi} [(-\cos \pi) - (-\cos 0)]$$

$$\frac{1}{\pi} [1 + 1] = \boxed{\frac{2}{\pi}}$$

$$6) \frac{1}{2e-e} \int_e^{2e} \frac{1}{x} dx$$

$$\frac{1}{e} \left[\ln x + C \right] \Big|_e^{2e}$$

$$\frac{1}{e} [\ln 2e - \ln e]$$

$$\frac{1}{e} [\ln 2 + \ln e - \ln e]$$

$$\boxed{\frac{\ln 2}{e}}$$

$$7) \frac{1}{3} \int_{-1}^2 [3x^2 + 2x] dx$$

$$\frac{1}{3} \left[x^3 + x^2 + C \right] \Big|_{-1}^2$$

$$\frac{1}{3} [(8+4) - (-1+1)] = \boxed{4}$$

$$8) \frac{1}{i} \int_0^1 \frac{1}{1+x^2} dx$$

$$[\arctan x + C] \Big|_0^1$$

$$\arctan 1 - \arctan 0$$

$$\boxed{\frac{\pi}{4}}$$